Problem Set Solutions

1. Make a conversion table from Decimal (Base 10) to Binary (Base 2) from zero to thirty-five. Zero fill the binary numbers to generate six bits.

NON-ZERO FILLED

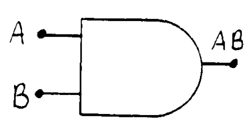
Decimal	Binary
00	0
01	1
02	1 0
03	1 1
04	1 0 0
05	1 0 1
06	1 1 0
07	1 1 1
08	1 0 0 0
09	1 0 0 1
10	1 0 1 0
11	1 0 1 1
12	1 1 0 0
13	1 1 0 1
14	1 1 1 0
15	1 1 1 1
16	1 0 0 0 0
17	1 0 0 0 1
18	1 0 0 1 0
19	1 0 0 1 1
20	1 0 1 0 0
21	1 0 1 0 1
22	1 0 1 1 0
23	1 0 1 1 1
24	1 1 0 0 0
25	1 1 0 0 1
26	1 1 0 1 0
27	1 1 0 1 1
28	1 1 1 0 0
29	1 1 1 0 1
30	1 1 1 1 0
31	1 1 1 1 1
32	1 0 0 0 0 0
33	1 0 0 0 0 1
34	1 0 0 0 1 0
35	1 0 0 0 1 1

ZERO FILLED

Decimal	Binary
00	0 0 0 0 0 0
01	0 0 0 0 0 1
02	0 0 0 0 1 0
03	0 0 0 0 1 1
04	0 0 0 1 0 0
05	0 0 0 1 0 1
06	0 0 0 1 1 0
07	0 0 0 1 1 1
08	0 0 1 0 0 0
09	0 0 1 0 0 1
10	0 0 1 0 1 0
11	0 0 1 0 1 1
12	0 0 1 1 0 0
13	0 0 1 1 0 1
14	0 0 1 1 1 0
15	0 0 1 1 1 1
16	0 1 0 0 0 0
17	0 1 0 0 0 1
18	0 1 0 0 1 0
19	0 1 0 0 1 1
20	0 1 0 1 0 0
21	0 1 0 1 0 1
22	0 1 0 1 1 0
23	0 1 0 1 1 1
24	0 1 1 0 0 0
25	0 1 1 0 0 1
26	0 1 1 0 1 0
27	0 1 1 0 1 1
28	0 1 1 1 0 0
29	0 1 1 1 0 1
30	0 1 1 1 1 0
31	0 1 1 1 1 1
32	1 0 0 0 0 0
33	1 0 0 0 0 1
34	1 0 0 0 1 0
35	1 0 0 0 1 1

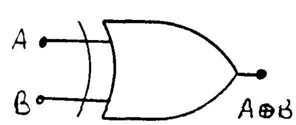
2. Draw the following gates and construct the truth tables for these gates.

2a)



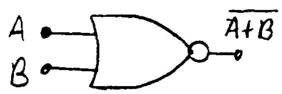
A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

2b)



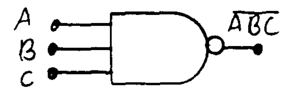
A	B	A⊕B
0	0	0
0	1	1
1	0	1
1	1	0

2c)



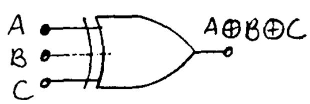
A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

2d)



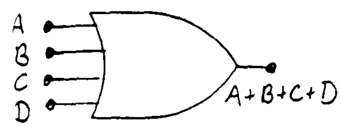
A	B	C	$\overline{ABC}$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

2e)



A	B	C	$A \oplus B \oplus C$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

2f)

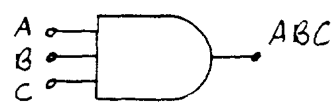


A	B	C	D	A+B+C+D
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

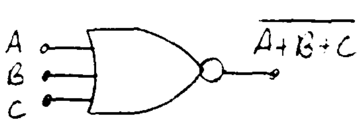
3. Draw the gates which represent the following Boolean symbols.

3a)

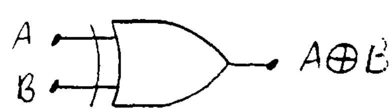
3b)



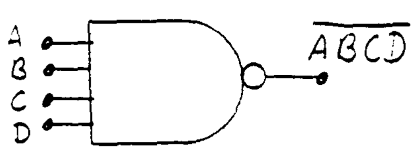
3c)



3d)



3e)



4. Prove the following Boolean Identities using Truth Tables.

4a) $0A = 0$

A	0	$0A$
0	0	0
1	0	0

Answer

4c) $AA = A$

A	A	AA
0	0	0
1	1	1

Verifies

4e) $(A+B)+C = A+(B+C)$

A	B	C	$A+B$	$(A+B)+C$	$B+C$	$A+(B+C)$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Verifies

4f) $A(\bar{A}+B) = AB$

A	B	$\bar{A}$	$\bar{A}+B$	$A(\bar{A}+B)$	AB
0	0	1	1	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	1	0	1	1	1

Verifies

4g) $A+BC = (A+B)(A+C)$

A	B	C	BC	$A+BC$	$A+B$	$A+C$	$(A+B)(A+C)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

Verifies

5. Prove the following equivalencies using Boolean algebra.

5a) $(A+B)(A+C)$

$= AA + AC + AB + BC$	Expand Out (Distrib OR)
$= A + AC + AB + BC$	Idempotent AND Law
$= 1A + AC + AB + BC$	Identity AND Law
$= A(1+C) + AB + BC$	Distributive OR Law
$= A(1) + AB + BC$	Null OR Law
$= A(1+B) + BC$	Distributive OR Law
$= A(1) + BC$	Null OR Law
$= A + BC$	Identity AND Law

Therefore $A + BC = (A+B)(A+C)$

5b) $A(\bar{A}+B)$

$= A\bar{A} + AB$	Distributive OR Law
$= 0 + AB$	Inverse AND Law
$= AB$	Identity OR Law

Therefore $A(\bar{A}+B) = AB$

5c) $(A + \overline{B+C})\bar{A}$

$= A\bar{A} + \overline{B+C}\,\bar{A}$	Distributive OR Law
$= 0 + \overline{B+C}\,\bar{A}$	Inverse AND Law
$= \bar{A}\,\overline{B+C}$	Commutative AND Law
$= \bar{A}\,\bar{B}\,\bar{C}$	De Morgan's OR Law
$= \overline{A+B+C}$	De Morgan's OR Law

5d) $AB + C + \overline{AB}CD + CC$

$= AB + C + \overline{AB}CD + C$	Idempotent AND Law
$= AB + \overline{AB}\,CD + C$	Idempotent OR Law
$= K + \bar{K}\,CD + C$	Substitution $K = AB$
$= K + CD + C$	Inclusion OR Law
$= AB + CD + C$	Substitution $AB = K$
$= AB + C$	Absorption OR Law

5e) $\overline{A\ \overline{B(B+C)}}$

$= \bar{A} + \overline{\overline{B(B+C)}}$	DeMorgan's AND Law
$= \bar{A} + B(B+C)$	Double Inversion
$= \bar{A} + B$	Absorption AND Law

5f) $A\bar{B} + A\bar{C} + BC$

$= A(\bar{B}+\bar{C}) + BC$	Destributive OR Law
$= A\,\overline{BC} + BC$	DeMorgan's AND Law
$= A + BC$	Inclusion OR Law

5g) $\overline{\bar{A}\,\bar{B}\,\bar{A}\,\bar{C}}$

$= \overline{\bar{A}\,\bar{B}\,\bar{C}}$	Idempotent AND Law
$= \overline{\overline{A+B+C}}$	DeMorgan's OR Law
$= A+B+C$	Double Inversion

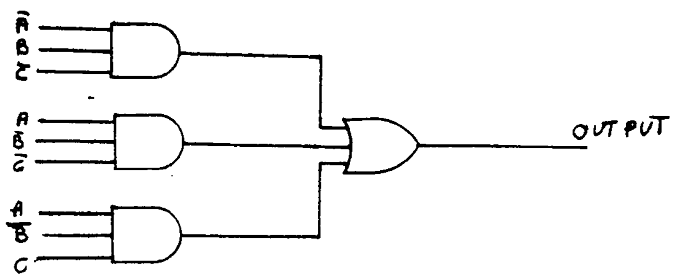
5h) $\overline{(\bar{A} + \bar{B} + \bar{C})C}$

$= \overline{\overline{ABC}}+\bar{C}$	De Morgan's AND Law
$= ABC + \bar{C}$	Double Inversion
$= AB + \bar{C}$	Inclusion OR Law

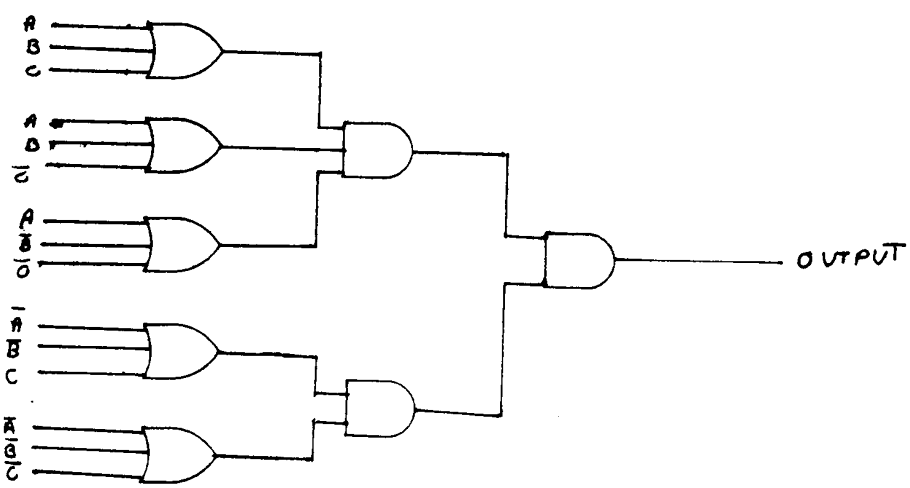
6. Write both sum of products and product of sums Boolean expressions for output Z using the truth table shown. Draw a logic diagram for both circuits.

Sop: $\bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$

B	C	Z
0	0	0
0	1	0
1	0	1
1	1	0
0	0	1
0	1	1
1	0	0
1	1	0

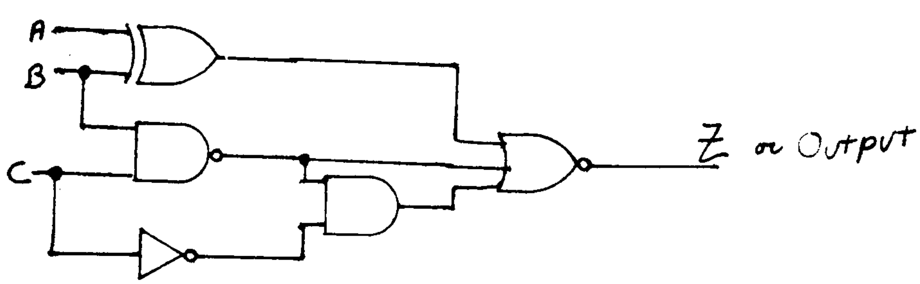


POS = $(A+B+C)(A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})$

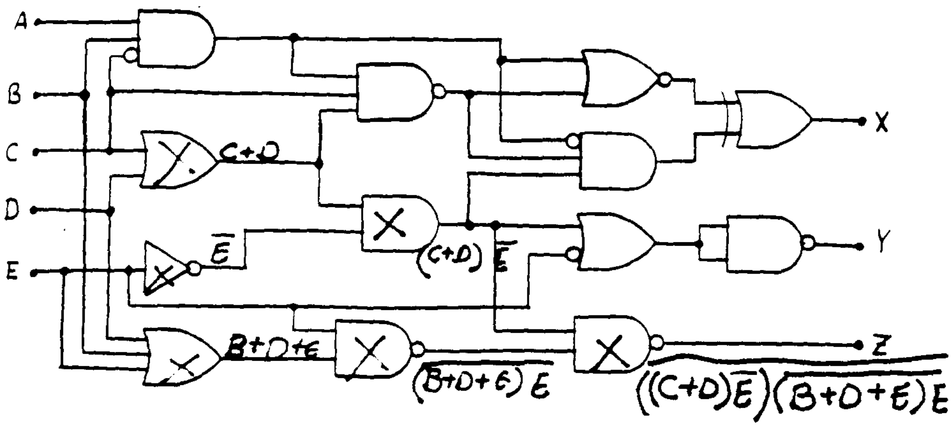


7. Draw a logic diagram for the following Boolean Expression.

$Z = \overline{A \oplus B + \overline{BC} + \overline{BC}\,\overline{C}}$



8. Write a Boolean Expression for Output Z.



$$Z = \overline{((C+D)\overline{E})\overline{(B+D+E)E}}$$

9. Construct the truth table for the 3 to 8 decoder of Figure 2.13a.

A	B	C	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

10.Construct a 4-bit adder using logic gates.